Teaching Math Skills While Teaching Elementary General Music

by Kirk Kassner, Ph. D.

Goethe thought of architecture as 'frozen music.' After nearly forty years of teaching music, I've begun to think of music as 'liquid math.' Because active music making integrates so many math concepts with musical skills, music teachers often implicitly reinforce what children learn in math class, just as we do in reading, social studies, science, and other disciplines. This article seeks to make those integrations explicit for two purposes: 1) demonstrate to math teachers how elementary general music can reinforce math learning, and 2) demonstrate to music teachers how they can help students better understand the relationships between music and math (MENC National Standard 8b. Identify ways in which the principles and subject matter of other disciplines taught in the school are interrelated with those of music.)

Just as MENC has established nine national standards and expectations for student learning in music (National Standards, 1994), the National Council of Teachers of Mathematics (NCTM) has established standards for mathematics (NCTM 2007) under three broad categories:

I. Understand numbers, ways of representing numbers, relationships among numbers, and number systems

II. Understand meanings of operations and how they relate to one another

III. Compute fluently and make reasonable estimates.

Each of these broad math categories is subdivided into distinct concepts and skills, as are the music national standards. NCTM encourages the use of music to help children learn math: "Adults can provide access . . . to music with actions and directions, such as up, down, in, out . . .. Children need things to count, sort, compare, match, put together, and take apart" (Standards K-2, p. 74), and "Schools should furnish materials that allow students to continue learning mathematics through counting, measuring, . . . and engaging in dramatic play, music, and art" (ibid, p. 75).

Below are illustrations of how some of the math subdivisions relate to music standards in kindergarten through grade two, and some sample music activities that serve learning simultaneously in both music and math. The primary purpose of each music lesson is to teach music skills and concepts, with math learning occuring in the course of teaching music. Music is front and center, not background or dispensable wall paper in service to math content. Thus, these illustrations follow the advice of Barrett, McCoy, and Veblen (1997): "teaching or learning music is dependent upon engagement *in music* [emphasis original]. This means that active music making is the highest priority for both teachers and students" (p. 36). Music is not used merely in service to math, but is taught for its own sake.

I. UNDERSTAND NUMBERS, WAYS OF REPRESENTING NUMBERS, RELATIONSHIPS AMONG NUMBERS, AND NUMBER SYSTEMS

Math subdivision:
COUNT WITH UNDERSTANDING AND RECOGNIZE "HOW MANY" IN SETS OF OBJECTS

Music standards: 1. Sing, 2. Play, 3. Improvise, 4. Compose, 5. Notate, 6. Listen, 7. Evaluate, 8. Connect with other subjects, 9. Connect with History/Culture

Sample music activities, which teach both music and math concepts

1. Sing *The Counting Song, Alison's Camel, Ten in the Bed, Brush Your Teeth, Four Songs* (Aden Lewis), *Johnny Works with One Hammer*
2. Play *Frere Jacque (Are You Sleeping), Rain Rain Go Away*
3. Improvise patterns for set number of beats, everyone else echos
4. Compose with specified number of beats, pitches, etc.
5. Read Ta's and TiTi's, count beats & rhythm patterns, such as in *The Sleeping Princess*. Count the number of sol's, mi's, etc. count beats in each measure or phrase of simplified written music, such as *Rain Rain Go Away*.
6. While listening to musical compositions, count beats in phrases, i.e., *William Tell Overture, Für Elise.* Show the number of beats in meter patterns by making different movements for each beat with "fun sticks" (12-inch sections of foam pipe insulation wrapped with duct tape). Start with teacher leading, then have students lead.
7. Use numbers to rate music and music performances on different characteristics
8. Explicitly explain/demonstrate the need to count when making music
9. Sing counting songs in different languages, i.e., *The Counting Song, Un Elephante*. Also, teach children to use American Sign Language when counting, and count in languages other than English.

Math subdivision:
USE MULTIPLE MODELS TO DEVELOP INITIAL UNDERSTANDINGS OF PLACE VALUE AND THE BASE-TEN NUMBER SYSTEM

Music standards: 1. Sing, 2. Play, 3. Improvise, 4. Compose, 5. Notate, 6. Listen, 7. Evaluate, 8. Connect with other subjects, 9. Connect with History/Culture

Sample music activities, which teach both music and math concepts

1. Sing *Ten in the Bed, Ants Go Marching, This Old Man, The Elephant Song, 1-2-3 Alary, Children Go Where I Send Thee*
2. Play *This Old Man* on barred instruments
3. Improvise with 10 beats, 20 beats, etc.
4. Compose in 5 meter
5. Read Ta's and TiTi's, count beats & rhythm patterns, use tally system to keep track of counting (circle two groups of 5 to show 10)
6. Listen to music compositions and keep track of beats using tally system above

7. Evaluate music and performance using 10 fingers to indicate base-10 Olympic scale
8. Explain that our number system is based on 10 fingers, but music often groups itself in multiples of 4, 8, 16, 32, etc.
9. Sing counting songs in different languages, i.e., *Un elephante, The Counting Song*

Math subdivision:

DEVELOP UNDERSTANDING OF THE RELATIVE POSITION AND MAGNITUDE OF WHOLE NUMBERS AND OF ORDINAL AND CARDINAL NUMBERS AND THEIR CONNECTIONS

Music standards: 1. Sing, 2. Play, 3. Improvise, 4. Compose, 5. Notate, 6. Listen, 7. Evaluate, 8. Connect with other subjects, 9. Connect with History/Culture

Sample music activities, which teach both music and math concepts

1. Sing songs with 1st, 2nd, 3rd, etc. verses, or contain a set sequence of ordinal events, i.e. *Nanny Goats Gruff, I'm in the Mood for Singing, Teddy Bear, I Know An Old Lady*
2. Play rondos with 1st, 2nd, 3rd, etc. phrases, and songs with 1st, 2nd, 3rd, etc. endings, play teacher-composed extra parts to Kodàly's *Viennese Musical Clock*
3. Improvise in sequences of students
4. Compose following a sequence of directions
5. Contrast rhythmic notation of phrases with different numbers of beats
6. Count beats in phrases while listening. Dance *Seven Jumps* for sequence of 1st, 2nd, 3rd, etc. Watch Silver Burdett Animated Listening Lessons of Chappelle's *The Add-On Machine*, and also *The Little Red Hen.*
7. Rate a list of songs or performances, resulting in 1st place, 2nd place, etc.
8. Use ordinal numbers to discuss the best order to present songs in a concert
9. When singing verse-refrain songs in other languages, use the ordinal verse numbers in the same language

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Math subdivision:

UNDERSTAND AND REPRESENT COMMONLY USED FRACTIONS, SUCH AS 1/4, 1/3, AND 1/2

Music standards: 1. Sing, 2. Play, 3. Improvise, 4. Compose, 5. Notate, 6. Listen, 7. Evaluate, 8. Connect with other subjects

Sample music activities, which teach both music and math concepts

1. Sing songs with multi-syllable words, such as *Ding Dong Digidigi Dong* and *Tideo*, dividing beats and point out that in two-syllable words each syllable gets ½ the beat, in three-syllable words each syllable gets 1/3 the beat, etc. Also sing songs which help students remember money concepts (*Slicker Sam*)
2. Play songs with multi-syllable words as above, substituting clapping or playing different instruments for different words
3. Students improvise on drums patterns alternating between one sound per beat and two sounds

per beats, three-sounds per beat, etc.

4. Students compose patterns as in improvisation above and represent the patterns with colored math squares. Develop word lists on a topic with columns for 2-syllable words, 3-syllable words, 4-syllable words, etc., then write a poem and have children compose melody for it
5. Students read simplified rhythm notation and speak the rhythm using some system, i.e., Kodály ta, titi, triple-ti, tikatika, Gorden day, to-day, do-to-day, do-to-day-to, etc.
6. Listen for forms which divide into two parts (AB), three parts (AAB, ABB, ABA, ABC), etc. such as *William Tell Overture, Für Elise, Hungarian Dance #3*
7. When evaluating music or performance, have children indicate their evaluation by standing in groups who rate 1-2, 3-5, 6-8, 9-10. Announce the evaluation as a fraction of the whole class.
8. Help students make the connection that beats divide in half, thirds, fourths, etc., like other things (hours, money, pizzas, gallons).

II. UNDERSTAND MEANINGS OF OPERATIONS AND HOW THEY RELATE TO ONE ANOTHER.

Math subdivision:

UNDERSTAND THE EFFECTS OF ADDING AND SUBTRACTING WHOLE NUMBERS

Music standards: 1. Sing, 2. Play, 3. Improvise, 4. Compose, 5. Notate, 6. Listen, 7. Evaluate,8. Connect with other subjects

1. Sing *Five Little Chickadees, Ten in the Bed, Five Little Snowmen, An Elephant*
2. Play *Skin and Bones*, subtracting players each verse to get softer and softer
3. Improvise a rhythm pattern by one person, two echo, then four echo, etc.
4. Compose a one-measure ostinato, then compose a second that sounds good with the first, then a third
5. Read notation of 2-meter ta's in *Blue Bird* & have students conduct in 2-pattern, then read song in 3-meter & have students conduct, then 4-meter, then *Unsquare Dance* in 7-meter
6. Listen to compositions in 5 above and show understanding of meter by conducting correct pattern.
7. Evaluate the dynamic loudness on the stereo - play a piece at mark 1, 2, 3, etc then have students hold up fingers to guess where the knob is set. Transfer this activity to their music making.
8. Help students make the connection that adding numbers to musical patterns or subtracting them changes the sound and feel of the music in drastic ways

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Math subdivision:

UNDERSTAND SITUATIONS THAT ENTAIL MULTIPLICATIONS AND DIVISION, SUCH AS

EQUAL GROUPINGS OF OBJECTS AND SHARING EQUALLY

Music standards: 1. Sing, 2. Play, 4. Compose, 5. Notate, 6. Listen, 8. Connect with other subjects, 9. Connect with History/Culture

1. Sing a round, such as *Are You Sleeping* in parts: have students divided the class in two equal groups. Then sing in three parts and four parts, dividing the class accordingly.
2. Play a song in round as above.
4. Compose a melody of nine beats for only three pitches used an equal number of times. Children need to figure out how many times each pitch can be used.
5. Read notation with divided beats, prolonged beats
6. Listen to musical compositions to identify the meter, then count the number of measures in each phrase, multiply to find the total number of beats in the phrase
8. Explain explicitly that being able to multiply and divide is important in many aspects of music making, not just in math class.
9. Dance the *Fjäskern* from Sweden with groups of two people. How many groups will we have if there are 24 students in the class? 20 students? 21 students? Dance *Doon Gul Ge* from Korea, in which children need to get into circles of four students during part of the dance, into a whole-class circle, then back into small groups.

III. COMPUTE FLUENTLY AND MAKE REASONABLE ESTIMATES

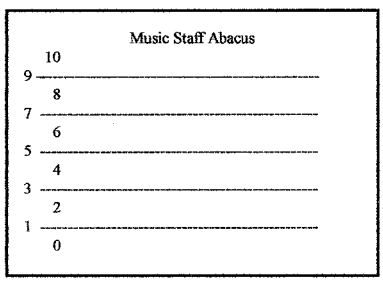
Math subdivision:
DEVELOP AND USE STRATEGIES FOR WHOLE-NUMBER COMPUTATIONS, WITH A FOCUS ON ADDITION AND SUBTRACTION

Music standards: 1. Sing, 2. Play, 3. Improvise, 4. Compose, 5. Notate, 6. Listen, 8. Connect with other subjects, 9. Connect with History/Culture

1. Sing and play *Five Fat Turkeys*. If the game starts with 20 turkeys and the cook catches and puts three in the oven, how many will be left – count remaining turkeys to double-check. Continue subtraction computations with different numbers of turkeys caught by the cook.
2. Teach students to play songs in 'chunks,' adding only one or two notes at a time.
3. Improvise with conga drums around a circle – first person starts with two-beat pattern and everyone echos, then second person plays the first person's pattern and adds two beats more improvisation and everyone echos. Continue adding two beats until someone can't play previous patterns correctly or add two beats, then start over. Challenge students to see how high a number of beats of improvisation they can get to.
4. As in improvising above, have one student write a two-pitch melody on the staff and sing it with the class echoing. Another student adds two more pitches and sings them, class echoing. Continue until writer or class is not able to sing the melody correctly, then start over.
5. As in improvising above, have one student write a two-beat pattern using ta, titi, and ta rest

notation, then clapping it and everyone echoing. Have another student add two more beats, clap, echo, then another student, etc. until the writer or class can no longer clap it correctly, then start over.

6. Listen to *Für Elise,* counting 123, 223, 323, 423, etc. giving students the strategy of representing measures as multiples of beats. Listening to Beethoven's *7th Symphony*, 2nd movement, explore the math and pattern as in Aden Lewis' *Hi Sister Susan* lesson.
8. Explain that we need to use math computations to perform and understand music better.
9. Introduce *Für Elise* with a time line of Beethoven's life from 1770-1827 – tell students about some major upheaval events in history and culture during this time period and ask them to figure out how old Beethoven was at each event [American Revolution, French Revolution, the decapitation of France's king and queen, the rise of Napoleon, the Louisiana Purchase, French conquering of Europe, Napoleon crowning himself Emperor, Beethoven changing the 3rd Symphony, Waterloo, etc.]



Music classroom management systems can also provide opportunities for math learning. By dividing the class into cooperative learning groups and using a music staff abacus to keep track of group points, the music teacher reinforces math concepts, while preparing children to learn music notation. The music staff abacus consists of a five-line standard music staff with a penny button, dime button, and dollar button for each group. Lines represent the odd numbers: 1, 3, 5, 7, 9. Spaces represent the even numbers: 0, 2, 4, 6, 8, 10. For example, placing the penny button on the middle line = 5, while the dime on the middle line = 50. When students earn points by answering questions or performing music, the teacher moves the penny button up, counting by line and space. When the penny button gets to the top (10) position, the penny button moves to the bottom (zero) position and the dime is raised one line or space to replace the ten pennies. Similarly, if the dime button reaches the top position, it moves to the bottom of the staff and the dollar button is advanced by one. Even kindergarten children can be taught to understand and use this system. When children learn later about composing numbers by place value and about the money system, they already have this concrete model of base-10 in their heads, and when they learn about placing note heads on a staff, the previous experiences with the staff abacus makes a natural bridge to pitch positions. For more ideas on motivation and management, see Chapter 11 of *Music in Childhood from Preschool Through the Elementary Grades, Third Edition* (2006).

Another classroom management system, which music teachers can use to reinforce math skills, is keeping a running total of class points, as in the Harry Potter books. In my school, each class is awarded 10 prize points every time they come to music class, and they keep all 10 points as long as everyone follows the class expectations: be kind, helpful, caring, and sharing; pay attention, try your best, and participate. Prize points are reduced if any student fails to meet expectations. At the end of each class, the prize points are added to the running total on the

class's prize chart. The class earns increasingly desirable prizes as their total reaches 150, 300, 450, 600, and 750. This system not only gives students an extrinsic motivation for high achievement in music, but also exercises their math knowledge. Their new total is announced at the end of each class and they are asked to calculate how many more points they need to get their next prize. Students always want to know which class has the most points, and they calculate the difference between classes.

Conclusion

The songs and music activities suggested above are but a small part of the huge repertoire of resources available to help children learn math while they learn music. In an ideal world, all teachers would have studied to earn endorsements in every subject, but I have not yet met anyone who had duel teaching endorsements in math and music. The next best thing would be for math and music teachers to discover more of what each other teaches and what methods and resources they use. One excellent place for math teachers to start this discovery is viewing the MENC web page on National Standards for Music at http://www.menc.org/publication/books/prek12st.html. Conversely, music teachers could take a few minutes and look at the NCTM web page standards at http://standards.nctm.org/document/appendix/numb.htm. Each set of standards explain the *why* and *what* that each discipline strives to teach, but do not specify *how* the subject should be taught to achieve the standards. For the *how* of teaching music, math teachers could look through teachers' manuals of a standard music basal textbook, such as Silver Burdett's *Making Music*. Music teachers could examine the teachers' manuals for a math basal textbook, such as . Another way to discover what each other teaches is for music teachers and math teachers to find time to talk with each other about what they are trying to accomplish at each grade level, and/or exchanging short-term and long-term goal lists and methods. I realize all these suggestions take time to implement in a setting where time is in short-supply and demands on that time are tremendous. But the rewards of linking our two disciplines are enormous for the children, help legitimize the role of music education in the overall curriculum, and help reduce the isolation often perceived of and felt by music educators within the larger school context. Music education will be stronger by demonstrating and making its correlations to math explicit, and math education will benefit from strong methods of re-teaching with musical resources.

Resources for music cited in this article:

MM = *Making Music* (Silver Burdett, 2005) *
MC = *Music Connection* (Silver Burdett, 1996) *
LLS = *Listen Look and Sing* by Aden Lewis (Silver Burdett, 1981) now out of print
BSTM = *Basic Skills Through Music (Misc)* by Aden Lewis (Alfred, 1981) out of print
TMD = *Teaching Movement and Dance* by Phyllis Weikart (High Scope, 1982)
MIC = *Music in Childhood from Preschool Through the Elementary Grades*, Third Edition (2006)
ALM = Animated Listening Maps (Silver Burdett, 2006) *

* Many of the selections cited can also be found in other basal texts, such as MacMillan's *Spotlight on Music*. The author simply lists products readily available to him. This does not constitute an endorsement or preference for one product over another.

Music Cited	Resource/Published by or in
Add-On Machine, The	AML CD 2-2
Alison's Camel	MM Grade K p. 289, CD 9-17
Are You Sleeping	MM Grade 2 p. 125, CD 5-2
Blue Bird	LLS Grade 2 p. 2, MM Grade K p. 105, CD 4-2
Brush Your Teeth	MM Grade 1 p. 272, CD 8-5
Children Go Where I Send Thee	MM Grade 3 p 382-3, CD 12-13
Clocks and Watches	LLS Grade 3 p. 57-58
Counting Song, The	MM Grade 1 p. 249, CD 7-12
Ding Dong Diggi Diggi Dong	MM Grade 3 p 14, CD 1-10
Doon Gul Ge	MM Grade 3 p. 130, CD 4-10
Five Little Chickadees	MM Grade K p. 179, CD 6-16
Five Fat Turkeys	LLS Grade 1, p. 29
Five Little Snowmen	MM Grade K p. 318, CD 10-14
Fjäskern	TMD p. 103, Record 2 side B
Frere Jacque	MM Grade 2, p 123, CD 4-20
Für Elise	MC Grade K p. 145, CD 3-40 or any market recording,
Hungarian Dance #3 (Brahms)	MM Grade 1 p. 208, CD 6-42
I Know An Old Lady	MM Grade 1 p. 366, CD 10-34
I'm in the Mood for Singing	MM Grade K p. 86, CD 3-28
Johnny Works with One Hammer	MIC p. 129, CD band 21
Little Red Hen, The	ALM CD 5-4
Nanny Goats Gruff	MC Grade 1 p. 178, CD 4-29

One Two Three Alary	MC Grade 2 p. 30, CD 1-29
Rain Rain Go Away	MM Grade 1 p. 54, CD 2-12
Seven Jumps	TMD p. 112 Record 2 side A
Skin and Bones	MM Grade 2 p. 362, CD 12-18
Sleeping Princess, The	LLS Grade 1 p. 14
Slicker Sam	BSTM, record band 6, side A
Symphony #7 (Beethoven)	MM Grade 1 p. 97, CD 3-23, LLS Grade 1 p. 19
Teddy Bear	MIC p. 130, CD band 22
Ten in the Bed (Roll Over)	MM Grade K p. 288, CD 9-16
Tideo	LLS Grade 3 p. 58-60, MM 2 CD 3-8
Troika	TMD p. 116-17, Record 2B band
Un Elephante	MM Grade 2 p. 158, CD 5-29
Unsquare Dance	MC Grade 7 p. 74, CD 3-14
Viennese Musical Clock	MC Grade 4 p. 16, CD 1-11
William Tell Overture	any market recording

Barrett, J. R., McCoy, C. W., & Veblen, K. K. [1997]. *Sound Ways of Knowing*. New York: Schirmer Books.

Campbell, P., Scott-Kassner, C., & Kassner, K. [2006]. *Music in Childhood from Preschool Through the Elementary Grades*, Third Edition. Belmont CA: Thomson Schirmer.

Consortium of National Arts Education Associations. [1994]. *National Standards for Arts Education*. Reston, VA: Music Educators National Conference. Also available on the internet: http://www.menc.org/publication/books/prek12st.html

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National Council of Teachers of Mathematics web page on April 2, 2007 at http://standards.nctm.org/document/appendix/numb.htm